Modeling Mediation

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0. Abstract. Research on political processes has made increasing use of formal models in recent years. Formal models help overcome vague causal rhetoric and facilitate the design of explanatory studies, thereby promoting the search for better solutions to political problems. Although formal models have been applied to bargaining, few of them deal with negotiation. Those that do rarely include intervenors. Models of intervention into disputes almost always assume that the intervenor is an arbitrator rather than a mediator, i.e. has decision-making power rather than just influence. And, if they deal with mediation, they confine their assumptions about cognitions and preferences to the bargainers rather than including the mediator. Like other forms of non-binding communication, mediation is still a subject of suggestive, but non-rigorous, theory. As a result, the empirical research on mediation has also suffered from a lack of integration and theoretical cumulativity; there is little systematic knowledge about how to measure, predict, or engineer the success of a mediation effort.

Recommendations for how to improve the quality of mediation research have emphasized the study of mediation tactics and contexts. However, there are reasons to prefer an emphasis on the strategy of mediation, especially if one wishes to draw strength from the formal modeling tradition of political analysis.

The strategic approach to the modeling of mediation is illustrated with a model of a conflict between two actors, called "Polluter" and "Victim". Pollution reduction has a gradually increasing cost for Polluter, and a gradually declining benefit for Victim, as it approaches total abatement. The conflict reduces itself to a question of how much one will pay the other once Polluter has eliminated the most efficient amount of pollution.

The two parties have some difficulty in reaching an agreement, because they have different beliefs about how well off they would be if they failed to agree and the decision were made by other means than negotiation. The closer their beliefs about this non-negotiated decision (called the "status quo"), the more likely they are to reach an agreement.

The third party is called "Mediator". His interest is in Polluter and Victim reaching an agreement. He can increase the probability of an agreement by influencing one or both parties to change their beliefs about the status quo. This influence costs Mediator resources. Mediator's goal is to maximize his "utility", which is defined as the benefit he gets from an agreement being reached multiplied by the probability that it will be reached, less any costs he has incurred to exercize influence over the other parties' beliefs about the status quo.

After the assumptions of the model are defined, some of their implications are drawn for Mediator's optimal decision about how much to spend on influence and how to allocate this amount between influence on the two parties. After a few general results, a numerical example is presented, and Mediator's decision problem is solved with a specific recommendation.

1. The State of the Art.

Students of politics since antiquity have used models to explain and prescribe political behavior. Plato's "Republic", Machiavelli's "Prince", Rousseau's "Social Contract", Marx's "Class", Weber's "Bureaucracy", Truman's "Group", and Easton's "System" are among the best known political models. A political model is a set of imaginary components and rules which are thought to correspond to some of the components and rules in the real political world, and which are expected to generate true or useful propositions about that world. Modeling, then, is as old as political science.

A new kind of modeling, however, has emerged especially since World War II: formal modeling. Formal models of politics differ from other models in the precision with which their rules are stated. This precision can be of two kinds: (1) a lack of ambiguity, and (2) the use of quantification. In the first case, the rules are unambiguous: they are clear enough that any person who applies them to the same situation can produce the same results. In the second case, the rules are quantitative: they have not only all-or-nothing or more-and-less forms, but quantitative forms, with precise amounts and rates.

For those concerned with practical political problems, both these kinds of formalization--disambiguation and quantification--are positive developments. Formal models, if applied to real situations, generate either precise predictions or precise prescriptions. The predictions can be tested against actual events, and the prescriptions can be carried out and their results evaluated. Experiments can be designed to help compare the predictions and prescriptions of competing models and to help select the model that receives the greater support. Formal models thereby assist in the solution of political problems.

Several political processes have been formally modeled. They include war, arms races, kidnapping, crime control, ethnolinguistic rivalry, electoral campaigning, and coalition formation. For those interested in mediation, the most relevant political processes go under the heading of "bargaining". Bargaining, broadly construed, includes all situations in which two or more actors negotiate before deciding what to do about how something of value is allocated and its allocation depends on the decisions they all make (Young, 1975, p. 5). Young (1975) divides formal models of bargaining into three groups: game-theoretic models, economic models, and manipulative models.

A review of formal models of bargaining reveals that they almost always simplify their imaginary rules in ways that make mediation irrelevant. Many models reduce the negotiation aspect of bargaining to an initial and accurate presentation to the bargainers of basic information about the bargaining situation: the number of bargainers, their decision alternatives, and their interests in one another's decisions. Those formal models that include further negotiation usually assume that no third parties are present or that third parties have no effect on the outcome. Indeed, the major formal models of the negotiation process itself (Bartos, 1974, pp. 75-100; Zartman, 1977, pp. 565-662) exclude intervenors from consideration.

What formal modeling of third-party intervention in bargaining exists is largely confined to situations in which the intervenor has the power to dictate the outcome. Let us use the term "arbitration" to designate such situations, including the special case of "final-offer arbitration", where the intervenor is required to select one of the bargainers' offers and impose it unchanged. Recently, a number of theorists have modeled situations in which an arbitrator is present and the parties are uncertain about one another's forthcoming proposals, about the arbitrator's preferences, or both (Ashenfelter and Bloom, 1981; Brams and Merrill, n.d.; Farber, 1980; Schotter, 1978).

Formal modeling has almost never been applied to the mediation of conflict, however, if we define "mediation" as a situation in which an intervenor can influence but not dictate the outcome. A recent comprehensive review of mediation research (Wall, 1981), though appearing in a journal noted for formal-model studies of conflict, does not mention a single study in which mediation is formally modeled.

The closest approximation to a formal model of mediation appears to be Cross's (1969) treatment of a kind of mediation that he calls "voluntary arbitration". This is a situation in which a neutral third party makes a proposal which the bargainers are free to accept or reject. Cross's model, in brief, posits that each bargainer expects the other to make concessions at a certain rate and adjusts that expectation as the other actually makes concessions. Since all proposals from bargainers, and only those, will be treated as concessions, proposals from third parties have different effects from identical proposals made by the bargainers themselves. In Cross's model, the bargainers treat a third-party proposal as a one-time opportunity to save additional negotiating costs. If they accept it, the bargaining ends; if not, the negotiation continues as if the arbitrator had never intervened. Cross's model allows one to determine when a rational bargainer who has certain expectations about the other bargainer's concessions and about future negotiation costs would prefer to accept a given third-party proposal rather than continue negotiating.

The model just mentioned permits a mediator to perform only one act: making a proposal. In reality, however, mediators use many different techniques to influence bargaining outcomes. Wall (1981) lists a total of 104 tactics that mediators are reported to use to help bargainers reach agreement. Clearly, Cross's model, though an important start, barely begins to cover the major aspects of mediation that have been reported in the literature.

The tendency of formal modelers to steer clear of mediation is understandable. Mediation involves communication, but it is traditional in the formal modeling of conflict to ignore communication among the parties or with others. The classical ("noncooperative") form of game theory assumes that the parties cannot communicate at all and have no way of reaching or enforcing binding agreements, even if it is in all their interests to do so.

Mediators have been commonly understood as using their communicative opportunities to manipulate the preferences, the information, and/or the reasoning of bargainers, so as to have one or the other of two effects: (1) bringing the bargainers to their senses--making them behave rationally--or (2) pressuring the bargainers into accepting an agreement that is not in their best interests--making them behave irrationally. But formal models of conflict typically assume that the parties have fixed preferences, perfect information about how well the possible outcomes would satisfy those preferences, constant (if not perfect) information about the impact of their possible strategies on the outcome, and a perfect willingness and ability to reason from their preferences and information to maximally preference-satisfying strategies. If a model assumes that the actors are and remain rational, it cannot incorporate the services of a mediator who works by increasing or decreasing their rationality.

The non-use of formal models has retarded progress in the understanding of mediation. Wall (1981) laments the "understudied, less than understood, and unrefined" state of mediation theory and suggests that there has been little if any theoretical analysis (not to mention formal modeling) of mediation in the last decade.

Since formal models typically lead to experimental tests, it is not surprising that the literature on the experimental study of mediation is also sparse. A recent survey of the field of bargaining behavior (Crott, Kutschker, and Lamm, 1977) notes that a few experimental studies of mediation have taken place, but summarizes (vol. 1, p. 148) the state of the art this way (my translation): ". . . mediation processes--and in particular why and under what conditions a mediator's proposal promotes agreement--have hardly been subjected yet to quantitative empirical research." The few experimental results that are reported test hypotheses from more/less or yes/no models, rather than quantitative ones. For example, concessions by bargainers were found in one experiment to be more generous when failure to agree by a certain deadline was going to call forth arbitration than when it was going to result in mediation (p. 150). The theoretically isolated experiments conducted so far yield, according to the authors (p. 152), "only the generalization that the relative effectiveness of conciliation, mediation, and other forms of intervention is dependent on the social characteristics of the bargaining situation."

Knowledge about mediation today, then, is largely deprived of the precision and theoretical integration that formal models have contributed to knowledge about other political processes. The storehouse of mediation wisdom is stocked with techniques but not with precise instructions for their application. Several of the 104 mediation tactics mentioned by Wall (1981) are pairs of opposites, e.g., "Clarifies situation" vs. "Deliberately misrepresents situation", or "Strengthens weaker side" vs. "Exploits weaker side". We know little about the conditions under which a tactic is more likely to be used--or be successful--than its opposite. Where, then, do we go from here?

2. An Approach to Modeling Mediation.

In looking for a way to model mediation, we might choose to focus on any of three aspects of it:

(1) the context of mediation;
(2) the tactics of mediation;
(3) the strategy of mediation.

The common response to the current unsatisfactory state of mediation research is to advocate a concentration on the first two aspects, and in particular on tactics _in_ context. "Under what conditions (situational and social factors, personality characteristics of the bargainers) is one technique of intervention more successful than another? There is a broad area here for empirical research" (Crott, Kutschker, and Lamm, 1977, vol. 1, p. 151). Wall (1981) likewise sets forth a research agenda oriented around the 104 mediation tactics he lists. He suggests investigating them in pairs to determine how potent they are in combinations. He further lists 19 contextual variables (e.g. the number and complexity of the issues being negotiated) and suggests that the efficacy of each tactic be investigated separately as a function of each contextual variable.

Given the large number of interesting tactical and contextual variables, there is a danger that any research agenda based on these will turn into an enormous fishing expedition. Wall's proposed agenda illustrates this danger well. By recommending that each tactic be paired with each other tactic for investigation, he is in fact proposing the study of 5,356 different pairs of tactics. If the impact of each contextual variable on each tactic were studied, 1,976 investigations would be required. If his two suggestions were combined, each of the more than five thousand pairs of tactics would be evaluated as a function of each of the nineteen contextual variables, requiring 101,764 research projects. And, if all possible combinations of the 104 tactics were studied rather than just two techniques at a time, the tactic-context approach would provide an agenda for over 385 nonillion (385,366,000,000,000,000,000,-000,000,000,000) studies!

The strategy of mediation is a more compact topic that is less likely to lure one into an aimless (and relatively fruitless) search for patterns. Strategic questions are also logically prior to tactical ones, since tactics are invoked as means to strategic ends. For example, before we can know whether the accurate or the distorted transmission of messages between the parties is a better tactic in mediation, we need to know what effects the transmission of messages is intended to bring about. A large number of tactics may be used to help produce a small number of effects. A strategic analysis would concentrate on these effects and assume that mediators have tactics by which they can--to a certain extent--produce certain desired effects. It would therefore ask what effects a mediator should be expected to want to produce and how the parties and the mediator should be expected to behave in view of the fact that they all have some power to affect the outcome of the bargaining.

A strategic focus is also more likely to help us tie mediation theory to formal models of conflict. Strategic behavior has been far more significant as a subject of formal modeling than has tactical behavior. Formal models of conflict typically assume that each party has a certain power to affect the outcome; they pay absolutely no attention to whence that power is derived, i.e. to skills, resources, and tactics. Although this is obviously a limitation of such formal models, it makes sense, when relying on this tradition, to capitalize on its particular strengths.

An example of emphasis on strategy is offered by Cross (1969, pp. 181-199), who explores to some extent the implications of his model for mediation. Cross assumes that a mediator may have influence over the bargainers' negotiating costs, costs of disagreement, discount rates (i.e. degrees of preference for sooner benefits to later ones), and learning rates (i.e. rates at which expectations get adjusted by experienced concession rates). The mediator may also have various goals, e.g. bringing about a faster agreement, bringing about a different agreement, bringing about a more durable agreement, or some combination of these. Cross shows how, if one accepts the assumptions of his model, a mediator with certain goals should prefer exercizing certain kinds of influence.

Cross's model is not, however, truly a model of mediation. The fundamental assumptions of the model--assumptions about the cognitive limits and the dynamically changing preferences of human actors--are applied only to the bargainers, not to the mediator. Mediator characteristics, such as degrees of influence, degrees of misestimation of bargainer characteristics, learning rates, and degrees of preference for various characteristics of an agreement, are not entered into the model. Thus Cross's model remains yet to be elaborated so as to make the mediator a full participant in the bargaining process.

A new approach to modeling mediation would be to recognize the mediator as a bargainer and attribute the same kinds of human qualities to the mediator as to the other bargainers. This approach will be illustrated below in a model that makes very few assumptions about the actors and therefore permits us to explore fully what the assumptions imply about the mediator's strategies and impacts.

3. An Example of Mediation Modeling.

Consider three actors, called "Polluter", "Victim", and "Mediator". Let us first follow the traditional practice and analyze the bargaining problem between Polluter and Victim, as if Mediator did not exist. Then we can bring Mediator into the picture.

Polluter is engaged in some activity that emits pollution. The pollution could be reduced or eliminated by the expenditure of money. As more and more units of pollution were eliminated, however, the cost per unit would go up, since the easiest-to-eliminate units would be attacked first. In mathematical terms, the unit cost of pollution reduction is "monotonically increasing".

Victim is hurt by the pollution that Polluter causes. Conversely, Victim would benefit from the its reduction or elimination. We can define the amount of benefit Victim would derive from a given reduction as the maximum amount of money Victim would be willing to give up in return for it. As more and more units of pollution were eliminated, however, Victim would be willing to give up less and less money per unit of reduction, since Victim would be cutting out more and more essential consumption in return for more and more inessential reductions of pollution. The unit benefit to Victim of pollution reduction is, in other words, monotonically decreasing.

These two assumptions about the trade-off between money and pollution for Polluter and Victim allow us to depict the relationship between these two actors, following Hamburger (1979, pp. 142-151), with a graph such as that in Figure 1. Curve C1 represents the possible levels of pollution, from the full unreduced level at "a" to the fully abated zero level at "z". Any point on C1 has an "x" coordinate, giving the monetary welfare of Polluter if Polluter pays for whatever pollution reduction takes place at that point. The "y" coordinate, or height, of the point represents the monetary welfare of Victim, defined as the amount Victim would be willing to pay to cause the given reduction of pollution from the full level. The assumptions about the increasing unit cost and the decreasing unit benefit of pollution reduction make C1 concave when viewed from below.

Initially, the bargaining problem consists of two questions:

(1) How much pollution will P be allowed to emit?

(2) How much money will P or V pay to the other?

If, however, we assume that Polluter and Victim both know the shape of C1, we can eliminate the first question from the bargaining problem by assuming that they will agree on "m" as the answer to that question. "m" is the amount of pollution for which the unit cost of reduction is equal to the unit monetary benefit of reduction. We assume Polluter and Victim will agree on "m" because, if they agreed on any other pollution level instead, they could improve <u>both</u> of their welfares by changing their agreement to "m". For example, if they were to agree on "r" instead of "m", they would attain welfare levels somewhere on C3, depending on how they answered question 2 above. But, for any point on C3, there are points on C2 that both bargainers would prefer. Any point on C2 that is above and to the right of the point on C3 would be preferred by both. And they could get to that preferred point by agreeing on "m" and on some answer to question 2.

This analysis leaves the parties with one bargaining problem: how much Polluter should pay to Victim, or Victim to Polluter, once Polluter has (at his own expense) reduced the level of pollution to "m". The solution to this problem depends on how much bargaining power each of the two bargainers has. In this model, this bargaining power is determined by one thing: how favorable the "status quo" is to each bargainer. The "status quo" refers to the situation that would exist if the two parties failed to agree on an answer to question 2 and settled their conflict through the non-bargaining institutions (e.g. courts or extortion gangs) available to them. The status quo can, like the possible bargaining settlements, be represented as a point on Figure 1.

One possible status quo is unreduced pollution with no payment between Polluter and Victim. This status quo would exist if there were a right to pollute. It would be represented by "a" in Figure 1, giving Polluter a welfare level of w[p](a) and Victim a welfare level of w[v](a).

Another possible status quo is completely reduced pollution (at Polluter's cost) with no payment between Polluter and Victim. This status quo would exist if there were a right to be free from pollution. It would be represented by "z" in Figure 1, giving Polluter and Victim welfare levels of w[p](z) and w[v](z), respectively.

There is an infinite number of other status quos. They can consist of various intermediate levels of pollution reduction without payment, or any level of pollution reduction with payment. In this model, it is assumed that all possible status quos involve no payment, but merely a specified level of pollution reduction paid for by Polluter. It is further assumed that both parties know that it is a no-payment status quo.

The model assumes that each party has some beliefs about what the status quo is. The beliefs of a party about the status quo take the form of a "probability distribution". That is, for any range of possible status quos, the party believes that there is a certain probability, between 0 and 1, that the actual status quo falls within that range. Any such range can be represented as a segment of curve C1 in Figure 1. If curve C1 is divided into some number of contiguous segments covering the entire curve, then the probabilities that the party assigns to those segments will add up to 1.

A special kind of belief situation that Polluter or Victim might be in is "certainty". In this situation, the party assigns the entire probability to one point on curve C1. From the party's point of view, he "knows" what the status quo is.

A particular special belief situation is one in which <u>both</u> parties are certain about the status quo. In such a case, we can represent each party's beliefs about the status quo by a single point on curve C1. Henceforth, this model makes the simplifying assumption that the situation is one of bilateral certainty about the status quo. This assumption is clearly unrealistic, but it can be defended by arguing that a party's probability distribution can be adequately summarized by a single point on C1. In other words, for any probability distribution, there is some certain status quo that the party would regard as equally advantageous or disadvantageous with that probability distribution.

Although the model assumes bilateral certainty about the status quo, it does not assume that the two parties have identical beliefs. They are both certain, but they do not necessarily agree. Thus one or both of them may have a false belief about what the status quo is.

First, however, we can define what the model predicts in the event that both parties do agree about the status quo. Then their beliefs are represented by one point on C1. As stated above, the model assumes the parties, if they agree at all, will agree on "m" as the actual level of pollution, and also on some payment between Polluter and Victim, so that the agreement can be represented by some point on curve C2. If both parties knew with certainty what the status quo is, would they agree to some point on C2 and, if so, what point would it be?

The model says that, if Polluter and Victim agreed about the status quo, they would come to an agreement about a point on C2 that gave each party at least the welfare level he would enjoy according to the status quo. No other agreement would be accepted because each party can independently guarantee to himself that level of welfare simply by refusing to reach an agreement with the other party. The points on C2 that satisfy this condition are all those points that are both to the right of the status quo and above the status quo. This set of points on C2 is called the "negotiation set". In the event that the parties agree about the status quo, the model says they will reach an agreement on some point in the negotiation set. The concept of a negotiation set is exemplified in Figure 2.

It might be objected that any model predicting a 100% probability of agreement is unrealistic. Even if the parties both have certain (and identical) knowledge about the status quo, they still have opposite preferences about the possible agreements within the negotiation set. To Polluter, the most preferred agreement in the negotiation set is the one at the lower-right end ("R" in Figure 2); to Victim, it is the one at the upper-left end ("T"). A number of theories and empirical observations, however, support the assumption that agreements are easy to reach in such situations, and they tend to cluster around the half-way point ("S") of the negotiation set (Cross, 1969). The model thus assumes an agreement when the parties are certain and identical in their beliefs about the status quo.

Now let us turn to the more general situation in which Polluter and Victim do not agree about what the status quo is. The parties' disagreement over the status quo implies that they also disagree over the negotiation set. Each party believes that the negotiation set is all the points on C2 that are above and to the right of where he believes the status quo is. Thus there are two perceived negotiation sets rather than one. At one extreme (the one just discussed), there is a total overlap between them. At the other extreme, there is no overlap. In between, there is partial overlap. The model will assume that, if the parties have different beliefs about the status quo, Polluter believes it is below and to the right of where Victim believes it is. In other words, each party has a belief about the status quo that is more favorable to himself than is the belief of the other party.

As discussed above, if there is 100% overlap between the two perceived negotiation sets, the model assumes that there is a 100% probability of agreement. If there is no overlap, the model must by the same logic assume that there is no probability of agreement, since there is then no point on C2 that both parties would prefer to their perceived status quos. Figure 3 illustrates this situation of no overlap. It shows that any point on C2 that is better for Victim than his perceived status quo, i.e. higher than w[v](s[v]), is worse for Polluter than Polluter's perceived status quo, i.e. to the left of w[p](s[p]). Thus one party or the other (or both) would always prefer non-agreement.

In the intermediate situation, there is partial overlap between the two perceived negotiation sets. As Figure 4 illustrates, this always implies that each negotiation set is partly inside and partly outside the other negotiation set. What should the model assume about the likelihood of agreement when the negotiation sets overlap partly but not completely? Three possibilities come to mind.

An optimistic assumption would be that any overlap at all is enough to assure an agreement. The basis for this is that, with even the slightest overlap, there is some agreement that would leave both parties better off than they would be under the status quos that they respectively believe to exist. In Figure 4, any agreement between T[p] and R[v] would have this feature.

A pessimistic assumption would be that an agreement will be reached only if there is total overlap between the two perceived negotiation sets. The basis for this is that any agreement reached without the benefit of total overlap would seem unfair to at least one party, since it would be detrimentally off-center on at least one party's perceived negotiation set.

A realistic assumption would be that if the negotiation sets partly overlap there is some probability, greater than 0 and less than 1, of the two parties reaching an agreement. Failing any good reason for a more complicated function, we could assume that this probability varies in a linear fashion with the overlapping proportion of the combined length of the two negotiation sets. In terms of the notation in Figures 1 and 4, if we quantify points on C2 by their distance from "A", the probability of agreement could be expressed as follows:

$$(1)\qquad L = \frac{2(T[p] - R[v])}{T[v] - R[v] + T[p] - R[p]} = \frac{2}{1 + \dfrac{T[v] - R[p]}{T[p] - R[v]}},$$

provided that $s[p] \leq s[v]$ and $T[p] > R[v]$.

Certain generalizations about the probability of agreement follow from the above assumptions. First, referring to Figure 1, we can describe each party's perceived status quo as being:

a. to the upper-left of "m";
b. at "m"; or
c. to the lower-right of "m".

It is clear that an agreement is possible only if both parties perceive that the status quo is of the same type (type "a", "b", or "c"). If their perceived status quos are not of the same type, their perceived negotiation sets will have no overlap.

Second, if both parties believe the status quo is at "m", they will have identical negotiation sets and will therefore reach agreement with a 100% probability.

Third, the probability of agreement can change if the perceived status quo of either party, or of both parties, changes so as to increase or decrease the proportion of overlap between their perceived negotiation sets. Under the optimistic assumption, this will happen when overlap comes into or goes out of existence. Under the pessimistic assumption, it will happen when overlap becomes or ceases to be total. And under the realistic assumption, it will happen when any change in the amount of overlap takes place.

Now we are ready to introduce Mediator into the model. Mediator, like Polluter and Victim, is assumed to be a purposive actor. Mediator has reasons (such as professional reputation and social esteem) for preferring that the bargaining between Polluter and Victim end in an agreement. Mediator also has some ability to influence the probability that they will reach an agreement. Although several strategies of influence might be imagined in mediation, our model assumes that only one strategy is available to Mediator. Mediator can cause one or both perceived status quos to move. By so doing, he can increase the overlapping proportion of the combined negotiation sets of Polluter and Victim and thus increase the probability of an agreement. We are not concerned with the many plausible techniques that Mediator might use to exercize his influence over the perceived status quos.

Mediator incurs costs for causing the perceived status quos to move. The costs could, concretely, consist of money, time, effort, or other resources expended either in the act of mediation or in preparation and training therefor. We shall, however, summarize these costs monetarily and call them "C". We shall also monetize the benefit Mediator obtains from an agreement being reached and call it "B". The model assumes that Mediator can evaluate any influence he is considering by determining the likelihood that an agreement would be reached if he exercized that influence. Let us call that likelihood "L". The "expected benefit" to him of exercizing the influence is the benefit of an agreement multiplied by its probability, or BL. Each influence alternative has a certain utility (called "U"), which is equal to its expected benefit reduced by its cost, or BL - C.

Mediator thus compares the utilities of all his influence alternatives, using U = BL - C as the measure. He thereby decides whether to exercize any influence and, if so, whether on Polluter, Victim, or both, and how far to move the status quo(s) which he decides to move.

The model assumes that the benefit of agreement to Mediator is constant, irrespective of the content of the agreement. Mediator does not care about whether the agreement benefits Polluter, benefits Victim, is fair, etc. He is also unconcerned--in this model--with any precedential effects of the agreement content on his difficulty in getting future bargainers to reach agreements. Thus "B" can be expressed by a single positive number. By definition, the benefit to Mediator from a failure of the bargainers to reach agreement is 0. Since the unit of measurement for cost and benefit is arbitrary, we can set this unit ourselves for our convenience. Let us use "B" as the unit of measurement, i.e. define Mediator's benefit from an agreement as equal to 1. This permits us to restate the utility of an influence alternative as U = L - C.

What determines the likelihood of agreement has already been described. Under the optimistic assumption, L = 1 if there is any overlap between the negotiation sets, and L = 0 otherwise. Under the pessimistic assumption, L = 1 if there is total overlap, and L = 0 otherwise. And under the realistic assumption, L = 0 if there is no overlap, and otherwise it has the value given by equation (1) above. Thus "L" will vary as a function of the influence that Mediator exercizes.

The cost of influence must now be defined. We can arbitrarily assume that if no influence is exercized then C = 0. Further, we shall assume that the cost of moving any party's perceived status quo is a monotonically increasing function of the distance of movement. Phenomena such as the "big lie", wherein major influence is allegedly easier to exercize than minor influence, are not recognized in the model. In addition, the model assumes that the cost of moving one party's perceived status quo a particular distance is independent of the amount of influence (if any) being exercized over the other party. Thus the model does not take account of the possibility that one party might be more--or less--willing to change his status quo if the other party is also changing his.

Mediator's guide for action has now been defined. All influence alternatives are to be compared, and the alternative offering the greatest utility (U = L - C) is to be selected. We shall now explore how Mediator can determine what this optimal alternative is.

If the two parties have identical beliefs about the status quo, Mediator's best course of action is to exercize no influence, since the utility of this alternative to Mediator is L - C = 1 - 0 = 1. Any influence would be superfluous, yet costly. Thus the solution to Mediator's problem is trivial unless the parties disagree about the status quo. Henceforth we assume they disagree.

Mediator's choice about whether and how to bring the parties' perceived status quos closer together is a complex one, because he is choosing two things at once: (1) how much to spend on influence, and (2) how to allocate that amount between the effort to influence Polluter and the effort to influence Victim. For each of these choices there is an infinite number of alternatives, resulting in a continuous two-dimensional decision space for Mediator. Hence he cannot literally carry out the above-mentioned requirement to compare all the alternatives.

Mediator can, however, reduce the complexity of his decision. First, if he can state mathematically the relationship between his costs of influence and the resulting overlap of the perceived negotiation sets, he may be able to find mathematically the optimal total cost and its optimal allocation. This result may be achieved by either analytical or numerical methods. Second, failing such a general solution, he may be able to determine some boundary conditions for the optimal decision. For example, there may be ways to determine whether any cost should be incurred at all, and, if so, for determining whether all of it should be devoted to influencing just one of the parties. And third, if Mediator's decision is subject to any external constraints, these may limit the decision enough so that Mediator can optimize it within those constraints. For example, Mediator may be prevented from spending more than a particular amount for influence (e.g. a bribery prohibition), or he may be required to allocate whatever he spends in a particular way between the parties (e.g. an evenhandedness requirement).

First let us see how far we can go in providing Mediator with a general solution to the decision problem. Before Mediator came onto the scene, a bargaining situation such as that shown in Figure 1 existed. This situation can be described by (a) the number of units of pollution being emitted by Polluter; (b) a function that relates Polluter's monetary welfare to the number of units of pollution reduction he carries out; (c) a function that relates Victim's monetary welfare to the number of units of pollution reduction Polluter carries out; (d) the status quo perceived by Polluter; and (e) the status quo perceived by.Victim. Let us denote these five characteristics of the situation by:

n = number of units of pollution
w[p](r) = Polluter's welfare with "r" units of reduction
w[v](r) = Victim's welfare with "r" units of reduction
s[p] = number of units of reduction Polluter believes is the status quo
s[v] = number of units of reduction Victim believes is the status quo

The likelihood ("L") that Polluter and Victim will reach an agreement depends on these characteristics in one of three ways. Whether the optimistic, the pessimistic, or the realistic assumption is an objective fact about the parties or a belief of Mediator is irrelevant to the model, but it would seem most sensible to attribute the assumption to Mediator. Thus, when we, for example, discover an "optimal" decision for a pessimistic Mediator, we shall mean that the decision would be optimal if Mediator's pessimism about "L" were correct. To reformulate "L" in terms of the above characteristics, we note that the "T" and "R" points on C2 can be described mathematically as follows:

$$T = \sqrt{2}\ \{w[p](A) - w[p](s)\}$$

$$R = \sqrt{2}\ \{w[v](s) - w[v](A)\}.$$

Without sacrificing generality, since we do not compare Polluter's and Victim's absolute welfares, we can arbitrarily set w[p](Z) = w[v](A) = 0 and w[p](A) = w[v](Z) = 1. Then

$$T = \sqrt{2}\ \{1 - w[p](s)\}$$

$$R = \sqrt{2}\ w[v](s).$$

We can also note that, when the two negotiation sets are just in contact, and thus $T[p] = R[v]$, it must be the case that $w[p](s[p]) + w[v](s[v]) = 1$. From the above it follows that, under the optimistic assumption, $L = 1$ if $w[p](s[p]) + w[v](s[v]) < 1$, and $L = 0$ otherwise; under the pessimistic assumption, $L = 1$ if $s[p] = s[v]$, and $L = 0$ otherwise; and under the realistic assumption,

$$(2) \qquad L = \cfrac{2}{1 + \cfrac{1 - w[p](s[v]) - w[v](s[p])}{1 - w[p](s[p]) - w[v](s[v])}}$$

if $w[p](s[p]) + w[v](s[v]) < 1$, and $L = 0$ otherwise. In all three cases, it is also assumed, as stated earlier, that $s[p] \leq s[v]$.

The second crucial piece of knowledge is Mediator's influence functions. These express the points to which Mediator can move the parties' perceived status quos with the expenditure of various resources. These functions can be called "f" and expressed as

$$s[p] = f[p](C[p])$$
$$s[v] = f[v](C[v]).$$

Given that the influence costs incurred by Mediator to influence the two parties add up to his total cost, we can also note that

$$C[p] + C[v] = C.$$

Substituting Mediator's influence cost functions for the status quo points that result from his influence into the expressions for "L", we arrive at

$$U = 1 - C \text{ if } w[p]\{f[p](C[p])\} + w[v]\{f[v](C[v])\} < 1$$
$$U = - C \text{ otherwise},$$

under the optimistic assumption;

$$U = 1 - C \text{ if } f[p](C[p]) = f[v](C[v])$$
$$U = - C \text{ otherwise},$$

under the pessimistic assumption; and

$$(3) \qquad U = \cfrac{2}{1 + \cfrac{1 - w[p]\{f[v](C[v])\} - w[v]\{f[p](C[p])\}}{1 - w[p]\{f[p](C[p])\} - w[v]\{f[v](C[v])\}}} - C$$

if $w[p]\{f[p](C[p])\} + w[v]\{f[v](C[v])\} < 1$ and

$U = - C$ otherwise,

under the realistic assumption. In all cases, it is assumed that

$$f[p](C[p]) \leq f[v](C[v]).$$

If this assumption is not met, Mediator has exercized excess influence beyond that required to guarantee an agreement.

Mediator's goal, then, is to maximize his utility, "U", which is a function of how much he spends on influence and how he allocates the expense between the two parties. In maximizing "U", he must abide by the following rules:

1. The amounts spent on influence, $C[p]$ and $C[v]$, must not be such as to make the status quo perceived by Polluter higher than the status quo perceived by Victim. (Such excess expenditure is useless, since the probability of an agreement cannot be raised to more than 1.)

2. If any amount is spent on influence, C[p] and C[v] must be such that there is some overlap between the perceived negotiation sets of the parties. (Any expenditure resulting in no overlap would be wasted, and thus its utility would be inferior to U(0,0).)
3. The sum of the influence expenditures must be less than 1. (If $C \geq 1$, then at best U = 1 - 1 = 0, which cannot be better than U(0,0).)

The optimistic and the pessimistic assumptions allow these rules to be made still more restrictive. Under the optimistic assumption, rules 1 and 2 are amended to read:

1o. If the initial negotiation sets overlap at all, Mediator must spend nothing on influence.
2o. If any amount is spent on influence, C[p] and C[v] must be such as to make the negotiation sets just begin to overlap. (We shall assume from here on that both "f" functions are smooth and that the cost of making the negotiation sets just barely overlap is the same as the cost of putting them just into contact.)

Under the pessimistic assumption, rule 2 is amended to read:

2p. If any amount is spent on influence, C[p] and C[v] must be such as to make the perceived status quos identical.

These rules delimit Mediator's decision space. Such a hypothetical decision space is shown in Figure 5. Each point in the space represents a pair of influence costs, C[p] and C[v]. The area bounded by the triangle results from rule (3) above. If the two initial negotiation sets are not in contact, a part of the triangle adjacent to the (0,0) corner will be excluded from the decision space by rule 2. It is the area from (but excluding) point (0,0) to (and including) curve L0; L0 represents the set of possible pairs of influence costs with a sum of less than 1 that would put the negotiation sets into bare contact. If there are ways to bring the two status quos to the same point at a cost of less than 1, another part of the triangle will be excluded from the decision space: the area to the upper-right of curve L1; L1 represents the pairs of influence expenditures with a sum of less than 1 that would make the status quos identical. The remaining area is the decision space under the realistic assumption. Under the optimistic assumption, however, the decision space consists of curve L0 only, if it exists, and otherwise just the point (0,0). Under the pessimistic assumption, the decision space consists of curve L1 only, if it exists, and otherwise just (0,0).

Each curve between L0 and L1 represents the ways Mediator could bring about a certain proportion of overlap between the perceived negotiation sets at a total cost of less than 1. For each proportion of overlap, one (C[p],C[v]) pair is the most efficient: the one with the minimum sum, "C". Since diagonals represent constant values of "C", the efficient pair of expenditures is given by the point on the curve that contacts the diagonal closest to the (0,0) corner. The set of efficient expenditure pairs takes the form of a transverse curve such as shown in the figure. Under the realistic assumption, Mediator's decision can be understood as a choice among the points on that curve. He will choose the point that gives the maximum value to U(C[p],C[v]). Under the optimistic assumption, the optimal decision will be at the intersection of the efficient curve and L0, or at (0,0) if L0 does not exist. Under the pessimistic assumption, it will be where the efficient curve intersects L1, if L1 exists, or otherwise at (0,0).

Although numerical methods will most likely be more economical than analytical methods for finding this maximum--if it can be found analytically at all--let us first see where analysis leads.

Under the optimistic assumption, the optimal decision is on L0 if L0 exists. L0 exists if there is at least one cost pair, (C[p],C[v]), such that

(4) w[p]{f[p](C[p])} + w[v]{f[v](C[v]) = 1

(5) C = C[p] + C[v] < 1.

Of the (C[p],C[v]) satisfying equations (4) and (5), the pair that minimizes "C" is the optimal decision. If no (C[p],C[v]) satisfies both equations, (0,0) is the optimal decision.

Under the pessimistic assumption, the optimal decision is on L1 if L1 exists. L1 exists if there is at least one cost pair, (C[p],C[v]), satisfying both equation (5) and

(6) f[p](C[p]) = f[v](C[v]).

Of the (C[p],C[v]) satisfying equations (5) and (6), the pair that minimizes "C" is the optimal decision. If no (C[p],C[v]) satisfies both equations, (0,0) is the optimal decision.

Under the realistic assumption, the optimal decision is in the area of the triangle bounded on the lower-left by L0 or the horizontal and vertical axes, and on the upper-right by L1 or the outer diagonal. That area exists if there is at least one cost pair, (C[p],C[v]), satisfying

(7) f[p](C[p]) ≤ f[v](C[v])

(8) w[p]{f[p](C[p]) + w[v]{f[v](C[v]) < 1

(9) U(C[p],C[v]) > 0,

with "U" defined by equation (3). Of the (C[p],C[v]) satisfying inequalities (7), (8), and (9), the pair that maximizes "U" is the optimal decision. If no (C[p],C[v]) satisfies all three inequalities, (0,0) is the optimal decision.

Finding the optimal decisions analytically according to the above rules is possible by straightforward procedures of algebra and differential calculus, if the "f" and "w" functions are smooth everywhere, although there is no guarantee that the differential equations that result will be soluble. For the optimistic version of the model, we can determine the endpoints of L0 by noting that at the upper-left endpoint either C = 1 or C[p] = 0, and at the lower-right endpoint either C = 1 or C[v] = 0. These possibilities lead to special cases of equation (4) that can be solved for C[p] or C[v]:

w[p]{f[p](C[p])} + w[v]{f[v](1 - C[p]) = 1

w[p](s[p]) + w[v]{f[v](C[v])} = 1

w[p]{f[p](C[p])} + w[v](s[v]) = 1.

If these equations yield two pairs of values for (C[p],C[v]) such that C ≤ 1, L0 exists and the optimal decision is at some point on it. That point is either an endpoint where C < 1 or an interior point, whichever has the smallest value of C = C[p] + C[v]. Computing "C" for the endpoints is trivial. Finding the minimum "C" among all the interior points requires reformulating equation (4) to express C[v] as a function of C[p], differentiating that function with respect to C[p], setting the derivative (or slope) of the function equal to -1, solving for C[p], and thence computing C[v]. If the optimal decision is at an interior point of L0, the values thus determined for C[p] and C[v] will be inside the triangle.

For the pessimistic version, the same requirements exist for the endpoints of L1. These lead to special cases of equation 6:

f[p](C[p]) = f[v](1 - C[p])

s[p] = f[v](C[v])

f[p](C[p]) = s[v].

The procedure from here is identical to that for the optimistic version.

For the realistic version, the procedure is analogous but more complex. The existence of L0 and L1 can be determined as above. From these results the shape of the decision space can be determined. Its bounds may include parts of the horizontal axis, the vertical axis, the diagonal, L0, and/or L1. The optimal decision may be a point on the horizontal axis, the vertical axis, or L1, or an interior point. Computing "U" for the endpoints of any of these bounds is no problem. The maximum "U" on the interior of L1 is at the same point as the minimum "C" on L1. The maximum "U" on the interior of the horizontal axis is determinable by substituting s[v] for f[v](C[v]) in equation (3), differentiating "U" with respect to C[p], setting this derivative equal to 0, and solving for C[p]. The maximum "U" on the interior of the vertical axis is determined analogously. Finally, the maximum "U" inside the bounds of the space will be a point where the partial derivatives of "U" with respect to C[p] and with respect to C[v] are both equal to 0. The maximum "U" over-all will be the greatest "U" resulting from these computations which is in the triangle.

The procedures summarized above may or may not yield solutions, depending on the forms of the "f" and "w" functions. If solutions do not emerge, Mediator can still use numeric methods, applying the above rules to various pairs of values for C[p] and C[v] and selecting the best pair among those tested. If the "f" and "w" functions are well-behaved, Mediator will be able to approach the actual optimum within any tolerance he may specify.

Short of an over-all solution, boundary conditions for the existence of certain kinds of solutions can still be checked. One such condition is for the expenditure of any influence resources at all. If it can be shown that Mediator can bring the two parties' perceived status quos completely together for less than 1 cost unit, then it follows that the optimal decision is one involving the exercize of Mediator influence. There are three ways Mediator might be able to bring the two status quos completely together. First, he might be able to move Polluter's status quo to where Victim's is. If Mediator can do this for less than 1 cost unit, it must be the case that

$$s[v] < f[p](1).$$

Second, Mediator may be able to move Victim's status quo to where Polluter's is for less than one cost unit. This is true if

$$s[p] > f[v](1).$$

Third, Mediator may be able to bring the status quos completely together for less than 1 cost unit by moving both of them toward each other. If so, there is some C[p] such that

$$f[p](C[p]) > f[v](1 - C[p]).$$

If Mediator can show that any of these three inequalities is true, he has proven that his optimal decision involves exercizing some influence over one or both parties.

The opposite kind of boundary condition would enable Mediator to know that his best decision is to leave the parties alone: to spend nothing on attempts to influence them. Clearly, this is the case if, by spending 1 cost unit, Mediator cannot bring their perceived negotiation sets close enough so they begin to overlap. An easy preliminary test for this condition is to determine how much overlap there would be if Mediator spent a full cost unit on influencing <u>each</u> party. If even this weren't enough to create any overlap, then obviously he could not produce any by dividing one unit between the two parties. If Mediator spent 1 cost unit on each party and their perceived negotiation sets were still non-overlapping, T[p] would be below R[v] (see Figure 3). This would mean that

$$w[p]\{f[p](1)\} + w[v]\{f[v](1)\} > 1.$$

Finally, there may be boundary conditions for the allocation of Mediator's influence expenditures between the parties. Under some conditions it may be possible to establish that all influence should be directed at just one party.

Suppose, for example, that Mediator is an adherent of the optimistic assumption, and that his influence functions are identical and linear. In other words, he believes that he can assure an agreement by getting the parties' perceived negotiation sets to just barely overlap, and he knows that he can move either party's status quo exactly "j" units per unit of influence cost he incurs, where "j" is the same for Polluter and Victim. Mediator's optimal decision will be to allocate his entire influence expenditure to one party.

To prove this, we can first note that, when the status quos are not in contact, w[p](s[p]) + w[v](s[v]) > 1. To bring them into contact, Mediator needs to change s[p] and/or s[v] so that w[p](s[p]) + w[v](s[v]) = 1. He can reduce w[p](s[p]) and w[v](s[v]) by any pair of amounts adding up to the initial difference between their sum and 1. Suppose first that both status quos are on the same side of "m". In that case, since C1 is concave from below and has a slope of -1 at "m", it must be true that above "m" a given change in "r" produces a greater change in w[p] than in w[v], and below "m" a given change in "r" produces a greater change in w[v] than in w[p]. Since both status quos are on the same side of "m" and will remain on the same side of "m" regardless of how they are brought together, it follows that it will always be more efficient to influence one party than the other if a given influence expenditure produces a constant and identical movement for either party. Above "m", Polluter should be influenced, and below "m" Victim should be influenced. Since Victim is always initially above Polluter on C1, that means the one closer to "m" should receive all the influence. (Inspection of Figure 3 helps make this plausible.)

Next suppose that the initial status quos are on opposite sides of "m". If Mediator first brings one of them to "m", then by the above reasoning he will want to continue moving the same party's status quo until his negotiation set contacts the other's. The only alternative is to bring both status quos together at "m", but this is an inferior alternative. It requires Mediator to move the status quos a total of s[v] - s[p] units. Moving one status quo until the negotiation sets meet requires less total movement, and hence less total cost, since elsewhere than at "m" the status quos do not need to coincide in order to make the negotiation sets come into contact.

It is of interest to observe that a different conclusion would be reached if Mediator were pessimistic rather than optimistic. Then he would need to bring the status quos to the same point, and this would require moving them a total of s[v] - s[p] units, regardless of how he did it.

It would seem plausible that this difference in optimal strategies between optimistic and pessimistic mediators would persist, even if with reduced force, when influence functions were nonlinear or nonidentical. Thus our model would lead us to expect, in similar bargaining situations, fundamentally different behaviors from the two kinds of mediators. Optimistic mediators (i.e. those who believe they can deemphasize the fairness issue and turn the possibility of a small mutual perceived gain into a settlement) would tend to work on the more central, or moderate, of the parties, trying to bring him closer to the more extreme belief of his adversary. Pessimistic mediators (i.e. those who do not think they can get an agreement until the parties see the world alike) would not have such a tendency. Processually, one could also predict that, if the average level of skill among the mediating profession rises, there will be an increasing tendency for mediators to encourage extreme beliefs rather than moderate beliefs among bargainers.

We have now discussed two of the three methods mentioned above for reducing the complexity of Mediator's decision: (1) an over-all solution and (2) the checking of boundary conditions. The most likely real mediation situation is one in which the influence and welfare functions are only partly known. Such partial knowledge may suffice to establish that certain boundary conditions are met, even though it prevents one from arriving at an over-all solution.

The third method named above was constrained optimization. If Mediator's decision is subject to constraints, he does not need the over-all solution discussed initially. We shall consider two kinds of constraint.

First, suppose Mediator is a realist and is constrained by a budget. He may spend no more than a certain amount on influence, even if spending more would give him a greater utility. If he knows that his constrained optimum is to use his entire budget on influence, he can simplify his problem to a single choice: how to allocate that amount between influence over the two parties. Let us call his budget "C". It can be represented graphically by one of the diagonal lines in Figure 5. The optimal ("U"-maximizing) point on that line is the one that contacts the outermost curve. It may lie at (C,0), (0,C), or an interior point. The value of U(C,0) is obtained by substituting s[v] for f[v](C[v]) and "C" for C[p] in equation (3). U(0,C) is computed analogously. If there is an internal maximum, the derivative of U(C[p], C - C[p]) with respect to C[p] will have to be 0.

Suppose, instead, that Mediator, still making the realistic assumption, is required to allocate any influence expenditure such that a certain proportion of it, "q", is spent on influencing Polluter. Then Mediator needs to find the "C" that will maximize "U" subject to this allocation constraint. This constraint is represented by a straight line passing through (0,0) in Figure 5. The lower "q" is, the steeper the line is. The maximum value of U(qC, [1 - q]C) may occur at (0,0) or at the outer endpoint of the line where f[p](qC) = f[v]([1 - q]C). On the interior of the line, "U" is defined by substituting qC for C[p] and [1 - q]C for C[v] in equation (3). If the derivative of U(qC, [1 - q]C) is everywhere measurable, it will be 0 at an interior maximum.

Having seen how, in principle, Mediator can find the optimal amount to spend on influencing each of two parties in a bargaining situation, let us consider a class of influence functions and a class of welfare functions that these methods could be applied to.

One class of influence functions takes the form

$$f[p](C[p]) = s[p] + j[p]C[p]^{1 - k[p]}$$

$$f[v](C[v]) = s[v] - j[v]C[v]^{1 - k[v]} ,$$

where "j" and "k" are constants. "j" indicates how much power Mediator has. It tells how many units he can move a party's perceived status quo by spending one unit on influence. "k" indicates the rate of power fall-off with increasing expenditures. When "k" is near 1, an increment of expenditure produces much less movement in the status quo than the previous equal increment of expenditure did. When "k" is near 0, an increment of expenditure produces close to the same amount of movement as the previous equal increment of expenditure. Because the Mediator's influence functions are assumed monotonically increasing, "j" must be positive and "k" must be less than 1. But "k" could be either positive or negative, reflecting either a marginally decreasing or a marginally increasing influence function. If "k" is negative, each successive equal increment of expenditure produces more movement in the status quo than the last.

Some examples of influence functions of the above form are given in Figure 6. The figure shows f[p] with six different combinations of values for j[p] and k[p]. In all the examples, "n" is 400 units of pollution, and Polluter's initial perceived status quo is 50 units of reduction. It can be appreciated from the figure that the two parameters of power ("j") and fall-off ("k") are enough to permit a wide variety of influence functions.

To formulate a class of welfare functions, we can first, referring to Figure 1, define the following terms:

$$g[p] = w[p](a) = w[p](0)$$
$$g[v] = w[v](z) = w[v](n).$$

As posited above, the possible welfare levels of the two parties range from 0 to 1, but their welfare levels under the possible status quos--which involve no payment--occupy only a part of that range. The greatest welfare a party could obtain from any status quo is defined here as "g": for Polluter, it is when the status quo is at point "a" (r = 0), and for Victim when it is at point "z" (r = n).

Consider welfare functions, then, of the following form:

$$w[p](r) = (1 - \frac{r}{n})\{-h[p]\frac{r}{n} + g[p]\}$$

$$w[v](r) = \frac{r}{n}\{(1 - \frac{r}{n})h[v] + g[v]\}.$$

These formulas contain a positive parameter "h", which indicates how nonlinear the function is. If "h" is near 0, the party's welfare function is almost linear, with each equal increment in pollution reduction either raising or lowering his welfare almost as much as the previous increment. If "h" is large, the function is very nonlinear, with successive equal increments of pollution reduction having much smaller impacts than previous ones. Whatever "h" is, the function ranges from 0 to "g" as "r" varies from 0 to "n" or vice-versa.

In addition to "h", we would also like to specify "m", the optimal number of units of pollution reduction. The values of "g" would then follow from the three specified values h[p], h[v], and "m", by virtue of two relationships. First, the slope of w[v] must be equal to the negative of the slope of w[p] at "m". Second, since "m" is on the diagonal, w[p](m) + w[v](m) = 1. Making use of these relationships in some manipulations that will not be shown here leads to the following results.

Once "m" has been specified, the necessity that g[p] + g[v] > 1 imposes an upper limit on the sum of the "h" parameters, which we shall call h[b]:

$$h[b] = h[p] + h[v] < \frac{2}{(m/n)^2 + (1 - m/n)^2 + 1}.$$

Once h[b] has been specified, it dictates the values of the "g" parameters. The monotonicity of each "w" makes g[p] and g[v] the upper limits on h[p] and h[v]:

$$h[p] < g[p] = 1 - (m/n)^2 h[b]$$

$$h[v] < g[v] = 1 - \{1 - (m/n)\}^2 h[b].$$

Specifying h[p] and h[v] within these limits permits us to reformulate w(r) solely in terms of "r" and the three specified parameters:

$$w[p](r) = (1 - \frac{r}{n})\{1 + \frac{r}{n}h[p] - (m/n)^2 h[b]\}$$

$$w[v](r) = \frac{r}{n}\{1 + (1 - \frac{r}{n})h[v] - (1 - m/n)^2 h[b]\}.$$

Two joint welfare functions with this form are shown in Figure 7. These examples are based on the following parameter values:

n = 100
m = 40
h[b] = 1.

The curves shown reflect two different pairs of values for h[p] and h[v]. In each curve, one dot represents one unit of pollution reduction. The density of "r" varies within each curve and between curves, depending on h[p] and h[v], so "m", though the 40th unit of "r" in each case, is not at a constant position in the welfare space.

Let us now seek a solution in a hypothetical case in which the influence and welfare functions have the above forms. To define the case, we assign arbitrary values to its parameters, as follows:

Let n = 100.
Let m = 65.

These parameters constrain h[b], as discussed above:

$$h[b] < \frac{2}{0.65^2 + 0.35^2 + 1} = 1.2945.$$

Let h[b] = 0.8.

This parameter determines g[p] and g[v] and constrains h[p] and h[v]:

h[p] < g[p] = 1 - (0.4225)(0.8) = 0.662
h[v] < g[v] = 1 - (0.1225)(0.8) = 0.902.
Let h[p] = 0.5.

This parameter determines h[v]:

h[v] = h[b] - h[p] = 0.3.
Let s[p] = 20.
Let s[v] = 70.
Let j[p] = 40.
Let j[v] = 60.
Let k[p] = 0.5.
Let k[v] = 0.2.

To begin the solution, we substitute these values for the parameters of the influence and welfare functions:

$$f[p](C[p]) = 20 + 40C[p]^{0.5}$$

$$f[v](C[v]) = 70 - 60C[v]^{0.8}$$

$$w[p](r) = (1 - \frac{r}{n})(0.5\frac{r}{n} + 0.662) = - 0.00005r^2 - 0.00162r + 0.662$$

$$w[v](r) = \frac{r}{n}(0.3 - 0.3\frac{r}{n} + 0.902) = - 0.00003r^2 + 0.01202r$$

We can now proceed with the solution, beginning with the (0,0) option, where Mediator exercizes no influence. Is there any initial overlap? Since the two parties' perceived status quos are on opposite sides of "m", there is none. Thus the expenditure of no resources would guarantee a non-agreement, and

$$U(0,0) = L - C = 0 - 0 = 0.$$

Now consider the horizontal edge, where Mediator exercizes influence only over Polluter. Is such a strategy worth considering? If Mediator spent 1 full unit of resources on influencing Polluter, he would move Polluter's perceived status quo to f[p](1), i.e. 60. Since that is still below "m", obviously there would still be no overlap. Thus the entire horizontal edge is excluded from the decision space.

How about the vertical edge? Spending a full unit on influencing Victim would move his status quo to f[v](1), i.e. 10. This is beyond Polluter's status quo. Thus Mediator can indeed improve upon (0,0) by some lower expense on the vertical edge. Under the pessimistic or realistic assumption, the vertical edge reaches the end of the decision space where C[v] would suffice to bring Victim's status quo exactly to where Polluter's is. The computations are:

$$f[v](C[v]) = s[p]$$

$$70 - 60C[v]^{0.8} = 20$$

$$C[v] = e^{(\ln 5/6)/0.8} = 0.7962$$

$$U(0,0.7962) = 1 - 0.7962 = 0.2038.$$

Let us assume henceforth that Mediator is a realist. Thus a possibly optimal decision is to spend 0.7962 on Victim, guaranteeing an agreement and giving Mediator a utility of 0.2038.

The vertical edge may contain an even better strategy, however. The decision space begins on the vertical edge where Victim's negotiation set just touches Polluter's, i.e. where $w[v]\{f[v](C[v])\} + w[p](s[p]) = 1$. This is solved for C[v]:

$$- 0.00003(70 - 60C[v]^{0.8})^{2} + 0.01202(70 - 60C[v]^{0.8})$$

$$- (0.00005)(400) - (0.00162)(20) + 0.662 = 1$$

$$0.108C[v]^{1.6} + 0.4692C[v]^{0.8} = 0.304$$

$$(0.3286C[v]^{0.8} + 0.7139)^{2} = 0.8137$$

$$\ln C[v] = (\ln 0.5727)/0.8 = - 0.6967$$

$$C[v] = e^{-0.6967} = 0.4982$$

Thus any (0,C[v]), $0.4982 < C[v] < 0.7962$, may produce a maximum of "U" on the vertical edge. Subjecting

$$U(0,C[v]) = \frac{2}{1 + \dfrac{0.18C[v]^{1.6} - 0.5172C[v]^{0.8} + 0.468}{0.108C[v]^{1.6} + 0.4692C[v]^{0.8} - 0.304}} - C[v]$$

to numerical maximization, however, reveals that the vertical-edge maximum in the decision space is at (0,0.7962), where total overlap is produced.

For the remaining points of the decision space we again use numerical methods. For each value in a systematic sample of values of C[p], U(C[p],C[v]) is computed for a systematic sample of values of C[v] within the decision space. The procedure is repeated for a smaller range of values of C[p] that the prior iteration revealed the maximum to be contained in. The forms taken by the influence and welfare functions result in rapid convergence to a maximum within the desired margin of error. The result is that the over-all maximum utility is achieved at an interior point of L1:

$$U_{max} = U(0.1333,0.5170) = 0.3497$$

$$f[p](0.1333) = 34.60$$

$$f[v](0.5170) = 34.60.$$

Mediator maximizes his utility by spending 0.65 on influence and allocating about one-fifth of that to influencing Polluter and the rest to influencing Victim. He moves Polluter's perceived status quo from 20 up to 34.60, and Victim's from 70 down to 34.60. This produces a certainty of agreement. By spending only 0.65 on influence rather than 0.8, which would have been required to achieve the same result by working only on Victim, Mediator gains a utility that is about 70% greater.

AUTHOR NOTE

This study was supported by the University of Washington Institute for Environmental Studies, under a grant from the Andrew W. Mellon Foundation. Additional support was provided by the Political Psychology Laboratory of the University of Washington.

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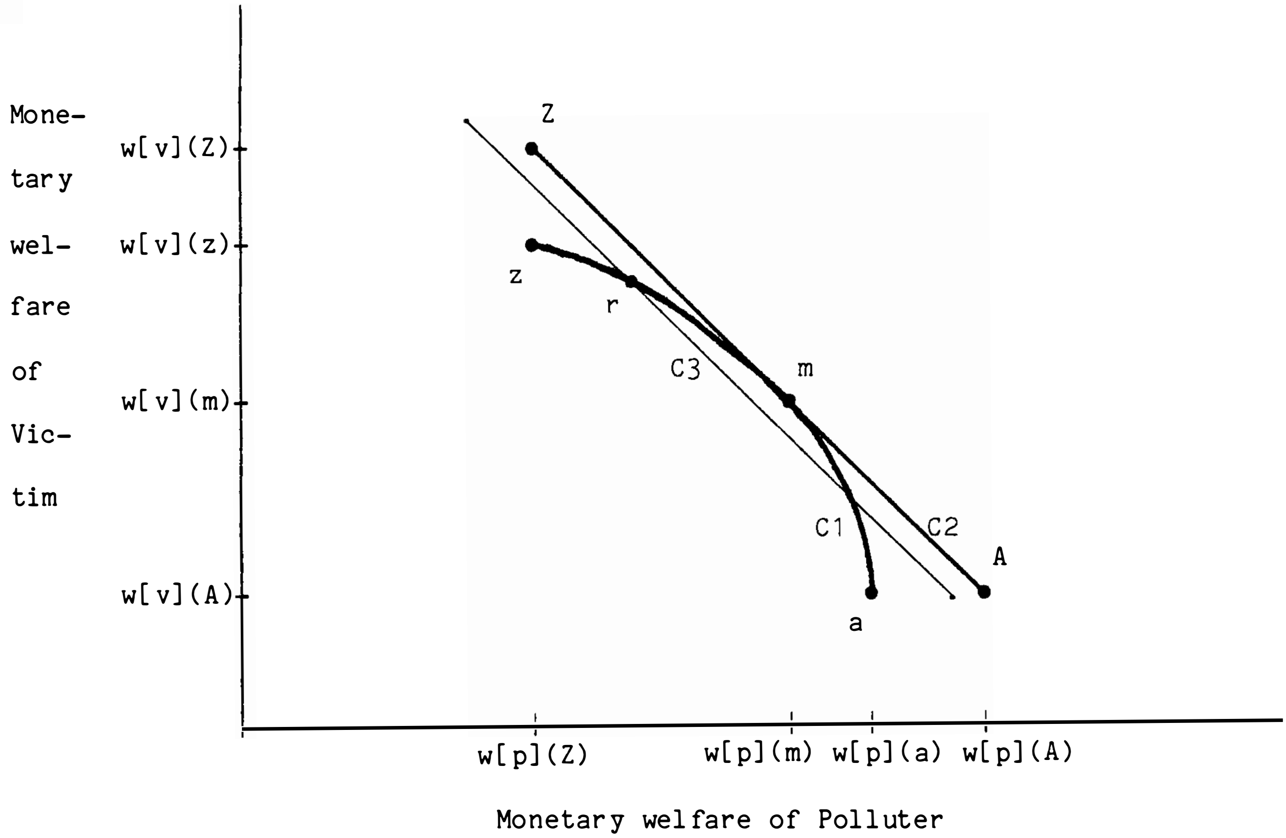

Figure 1. Relationship between two bargainers without mediation.

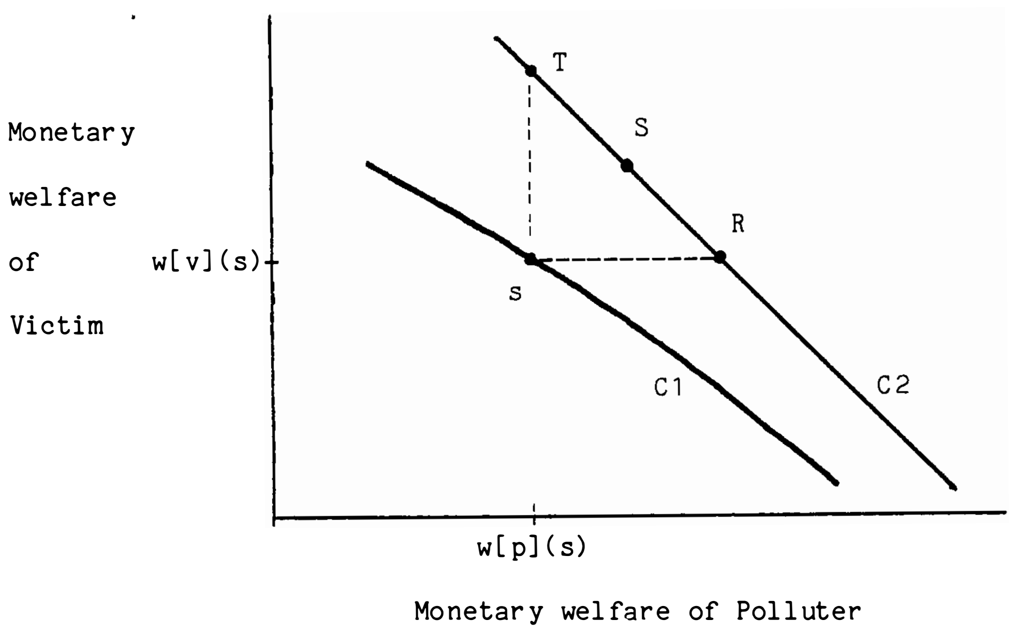

Figure 2. Agreement between two bargainers with identical bilateral certainty about the status quo

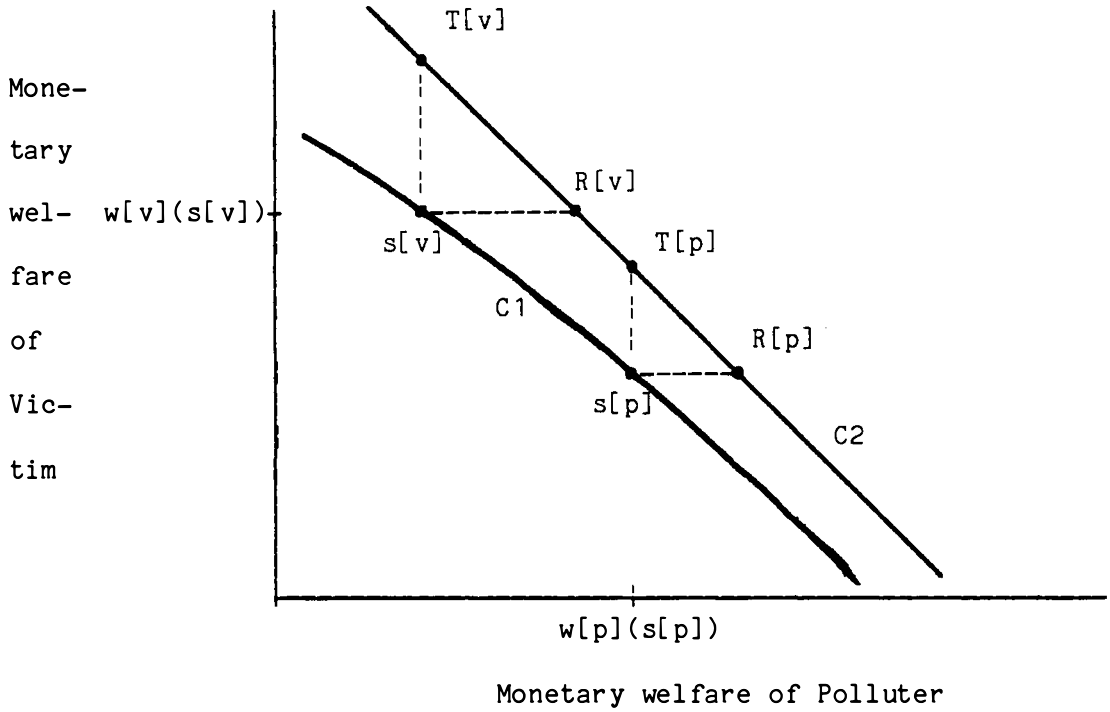

Figure 3. Disagreement between two bargainers with bilateral certainty about status quo and non-overlapping negotiation sets

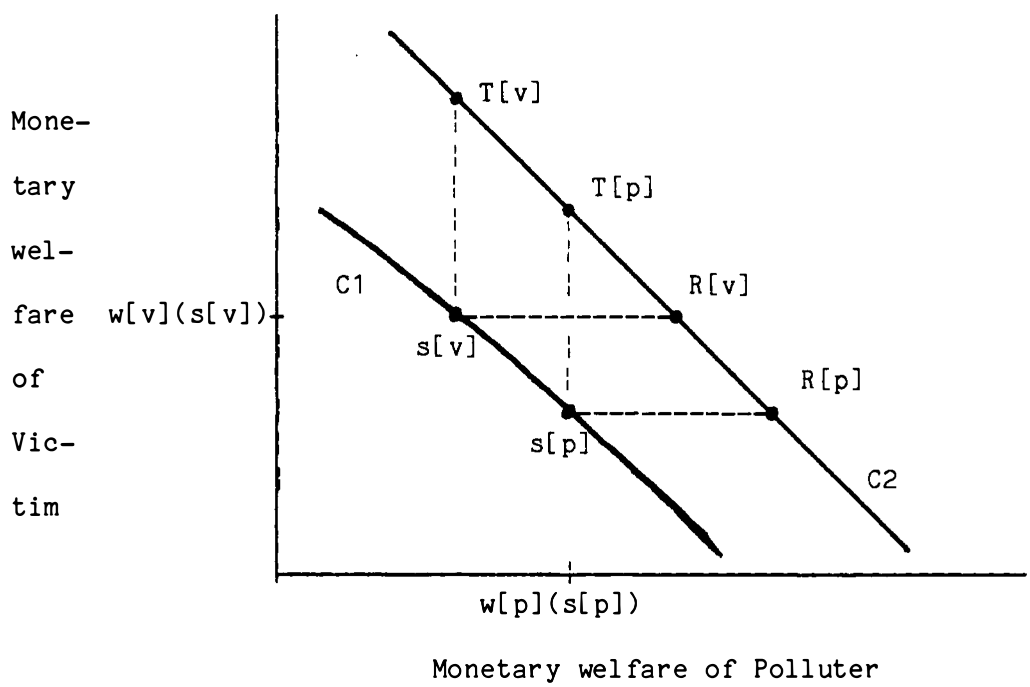

Figure 4. Possible agreement between bargainers with bilateral certainty about status quo and partially overlapping negotiation sets

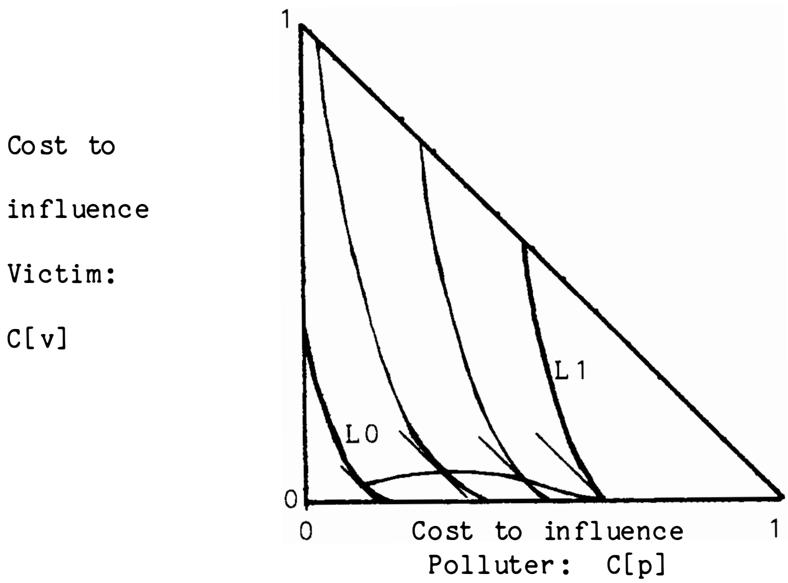

Figure 5. Mediator's decision space.

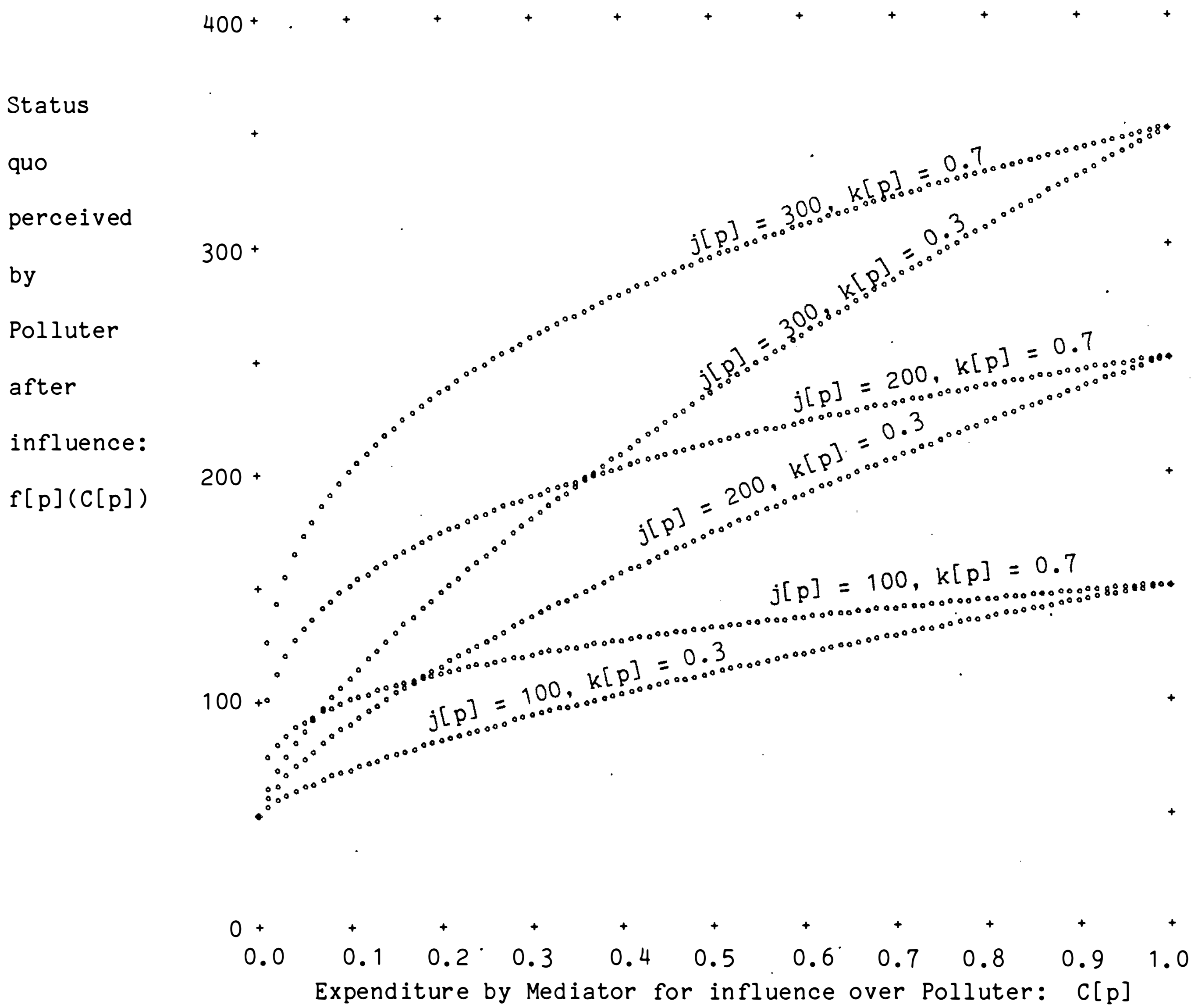

Figure 6. Examples of Mediator influence functions

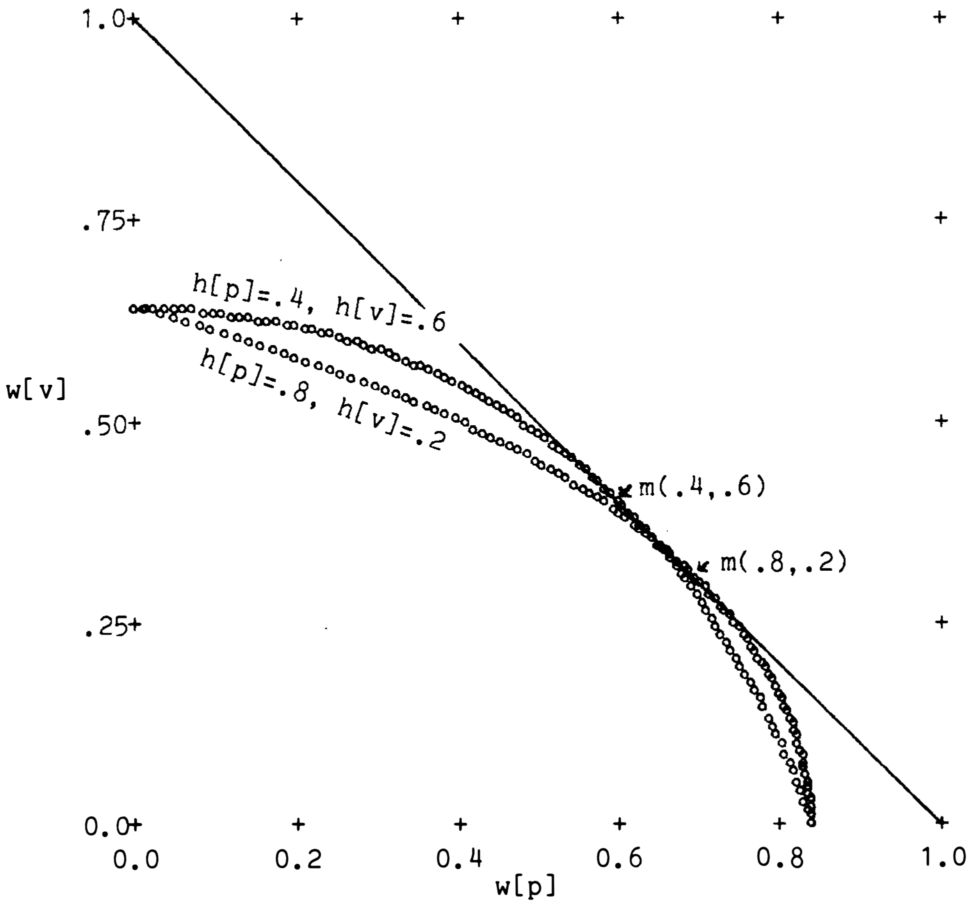

Figure 7. Examples of joint welfare functions

with m = 40 and n = 100